Journal of Material Science and Mechanical Engineering (JMSME)
Print ISSN: 2393-9095; Online ISSN: 2393-9109; Volume 2, Number 8; April-June, 2015 pp. 1-8

http://www.krishisanskriti.org/jmsme.html

A Computational Method for Non-Fourier Heat Transfer of Jeffrey's Type

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Abstract—*In this paper, a unique finite-differenced Flow field Dependent Variation (FDV) computational method is presented to solve non-classical heat transfer like Jeffrey's type non-Fourier heat conduction in still fluid or solid. By Taylor's series expansion strategy, the original Jeffrey's non-Fourier heat conduction equation is redeveloped into a form which now resembles the energy component of the conservative Navier-Stokes System of equations with zero flow velocity components. On applying the FDV methodology to the redeveloped Jeffrey's non-Fourier heat conduction equation leads to the FDV equation for Jeffrey's non-Fourier heat transfer which is numerically discretized by the appropriate finite differences schemes to obtain the final finite differenced FDV method for the numerical solution of non-Fourier heat transfer of Jeffrey's type in still fluid or solid . Numerical results based on our finite differenced FDV scheme for a one-dimensional Jeffrey's type non-Fourier heat conduction in a still nano-fluid demonstrated the capability of FDV method to solve the non-classical heat transfer also. Furthermore numerical results of the manner in which the Heat Conduction Model Number 'F_T' affects the non-Fourier temperature behavior showed agreement with the predecessor's published result of diffusion effects on non-Fourier temperature behavior due to increase in the values of 'F_T'.*

Keywords: *Flow field Dependent Variation theory, Navier-Stokes System of equations, non-Fourier, Heat waves, Second sound, thermal relaxation time, Jeffrey's heat conduction equation.*

1. INTRODUCTION

One of the important challenges in Computational Fluid Dynamics (CFD) is how to deal with very rapid changes of the solution variables like pressure, temperature, velocity and density both in time and space, where we are faced with smallest time and length scales for very high gradient variables. Further pairing challenges are the computational difficulties in resolving real complex flows as they are mixtures of physical phenomena like transition from laminar to turbulent flow, interactions between viscous & in-viscid flows, and incompressible & compressible flows. To tackle these challenges and resolve simultaneous all physical situations of fluid dynamics, Prof. T.J Chung and his co-workers [1-5] have introduced FDV method just at dusk of the 20th century. Since its inception, many benchmark cases [1-5] of fluid dynamics and classical heat transfer have been solved by FDV method to prove its excellent solution accuracy and numerical stability. Here we explore the possibilities of extending the benefits of FDV method to Computational Heat Transfer (CHT) by solving heat transfer phenomena influenced by both non-classical / non-Fourier and classical / Fourier heat transport effects and mechanism in a single domain. The classical heat flux constitutive model of the Fourier's type is a steady state type equation and it does not account for the very short transient time required to reach the steady state equation when a temperature gradient is suddenly applied. During such very short transient time, the heat conduction is non- Fourier in nature i.e. hyperbolic thermal waves / second sound propagation [6, 7] takes place and we have following [8] models.

Jeffrey's model : $\left(\frac{1}{a^2}\right)\frac{\partial^2 T}{\partial t^2} + \left(\frac{1}{\alpha}\right)\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \mathbf{K}\frac{\partial\left(\frac{\partial^2 T}{\partial x^2}\right)}{\partial t}$ (1)

Heat Conduction Model Number $F_T = \frac{\mathbf{K}}{\tau}$ was introduced by Tamma and Zhou [9] to improve the understanding relationships between the various heat flux constitutive models. For $\mathbf{K} \neq 0$ i.e. $0 < F_T < 1$, the thermal behavior is parabolic as $k_1 \neq 0$ and for $\mathbf{K} = 0$ i.e. $F_T = 0$, the thermal behavior is hyperbolic as $k_1 = 0$.

Cattaneo's model: If $k_1 = 0$ in Eq. (1), the Jeffrey's model

reduces $\left(\frac{1}{a^2}\right)\frac{\partial^2 T}{\partial t^2} + \left(\frac{1}{\alpha}\right)\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ (2)

and thermal behavior of is hyperbolic and the

transmission of heat is by thermal waves. Further If $\tau = 0$ in Eq. (2), the Cattaneo's model reduces to the well known classical Fourier's thermal conduction equation $\frac{\partial T}{\partial t} = \alpha\,\frac{\partial^2 T}{\partial x^2}$ and the thermal behavior is of parabolic and the transmission of heat is by diffusion. By Taylor's series expansion strategy [10], the Jeffrey's heat conduction Eq. (1) can be redeveloped into an equation form resembling the **energy component** of conservative Navier-Stokes System of Equations .The redeveloped Jeffrey's heat conduction equation is given respectively as:

$$\frac{\partial T(x,t+\tau)}{\partial t} = \alpha \frac{\partial^2 [T(x,t+\mathbf{K})]}{\partial x^2} \quad (3a)$$

$$\frac{\partial [\rho c T(x,t+\tau)]}{\partial t} + \frac{\partial \left\{\frac{\partial [-k\, T(x,t+\mathbf{K})]}{\partial x}\right\}}{\partial x} = 0 \quad (3b)$$

The energy component of the Navier-Stokes System of Equations for stand still constant property fluid condition (velocity v_1=u=0) without source terms for 1-D can be expressed in conservation form [5] as

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}_1}{\partial x_1} + \frac{\partial \mathbf{G}_1}{\partial x_1} = 0 \quad (4)$$

Where $\mathbf{U} = [\rho c T]$, $\mathbf{F}_1=[0]$ & $\mathbf{G_1} = [-kT_{,1}]$ (5)

$$\therefore \frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{G}_1}{\partial x_1} = 0 \quad (6a)$$

$$\frac{\partial [\rho c T]}{\partial t} + \frac{\partial [-k\mathbf{T}_{,1}]}{\partial x_1} = 0 \quad (6b)$$

The comma in expression $\mathbf{G}_1$ indicates partial derivative w.r.t independent variable x_1 (= x). Eq. (6b) is also the well known 1-D unsteady heat conduction equation in solids. On comparing Eq. (3b) & Eq. (6a), it shall be noted the form of the both equations are same. Further on writing, $\rho c T(x, t+\tau) = \mathbf{U}(x, t+\tau) = \mathbf{U}_\tau$

$$\& \frac{\partial [-k\, T(x,t+\mathbf{K})]}{\partial x} = \mathbf{G}(x, t+\mathbf{K}) = \mathbf{G_K} \quad (7)$$

we can thus transform the Eq. (1) into a form similar to Eq. (6a) [resulted from the Eq. (4)] through Eq. (3b). That is by Taylor's series expansion strategy as adopted in our earlier investigation [10], the original Jeffrey's heat conduction Eq.(1) can be redeveloped into such a single term L.H.S form Eq.(3a) which now resembles a case of the Navier-Stokes System of Equations which has the computational advantage of being numerically solved by the FDV method.

2. FINITE DIFFERENCED FDV EQUATION FOR NON-FOURIER HEAT TRANSFER OF JEFFREY'S TYPE

Based on Taylor's series expansion,

We denote $\mathbf{U}_\tau = \mathbf{U}(x, t+\tau) = \mathbf{U}(x, t) + \tau \frac{\partial \mathbf{U}(x,t)}{\partial t}$ &

$\mathbf{G_K} = \mathbf{G}(x, t+\mathbf{K}) = \mathbf{G}(x, t) + \mathbf{K} \frac{\partial \mathbf{G}(x,t)}{\partial t}$

& since $\rho c T(x, t+\tau) = \mathbf{U}(x, t+\tau)$

& $\frac{\partial [-k\, T(x,t+\mathbf{K})]}{\partial x} = \mathbf{G}(x, t+\mathbf{K})$, we can write the Eq. (3b) as

$$\frac{\partial \mathbf{U}_\tau}{\partial t} + \frac{\partial \mathbf{G_K}}{\partial x} = \mathbf{0} \text{ or } \frac{\partial \mathbf{U}_\tau^n}{\partial t} + \frac{\partial \mathbf{G}_\mathbf{K}^n}{\partial x} = \mathbf{0} \quad (8)$$

Based on the FDV methodology [5] , on expanding $\mathbf{U}_\tau^{n+1}$ in a special form of Taylor series about $\mathbf{U}_\tau^n$ up to and the second – order time derivatives and incorporating the appropriate first order (s_3) & second order (s_4) FDV diffusion parameters for the first and second order derivatives of $\mathbf{U}_\tau^n$ w.r.t time respectively, we present our derived FDV equation for one dimensional non-Fourier heat conduction of Jeffery's type in still constant property fluid or in solid as below with $O(\Delta t^3)$:

$$\Delta T_\tau^{n+1} = -\mathbf{E}_1^n \frac{\partial(\Delta T_\tau^{n+1})}{\partial x} - \mathbf{E}_{11}^n \frac{\partial^2(\Delta T_\tau^{n+1})}{\partial x^2} - \mathbf{Q}^n \quad (9a)$$

$$\Delta T_\tau^{n+1} + \mathbf{E}_1^n \frac{\partial(\Delta T_\tau^{n+1})}{\partial x} + \mathbf{E}_{11}^n \frac{\partial^2(\Delta T_\tau^{n+1})}{\partial x^2} = -\mathbf{Q}^n \quad (9b)$$

Where

$$\mathbf{E}_1^n = (\Delta t s_3\, \mathbf{b}_1)^n \quad (10a)$$

$$\mathbf{E}_{11}^n = \left(\Delta t s_3 \mathbf{c}_{11} - \frac{\Delta t^2}{2} s_4 (\mathbf{b}_1)^2\right)^n \quad (10b)$$

$$\mathbf{Q}^n = \frac{1}{\rho c}\left(\Delta t \frac{\partial \mathbf{G}_\mathbf{K}^n}{\partial x} - \frac{\Delta t^2}{2}\, \mathbf{b}_1^n \frac{\partial^2 (\mathbf{G}_\mathrm{K}^n)}{\partial x^2}\right) \quad (10c)$$

Finally resulted governing FDV equations Eq. (9) for 1D non-Fourier heat conduction is numerically discretized by FDM, FEM, or FVM [5] which is prerogative of Computational Fluid Dynamicists / Numerical Heat Transfer Analysts.

As exemplar, numerically discretizing FDV Eq. (9) by finite difference method (FDM) is illustrated next On approximating first order and second order spatial derivatives of Eq. (9) at each grid point (i-1, i, i+1) by second order accurate central finite differences, we present the final derived finite-differenced FDV equation Eq.(11) for one dimensional non-Fourier heat transfer of Jeffery's type in still constant property fluid or in solid as below with $O(\Delta x^2, \Delta t^3)$:

$$(\Delta T_\tau)_i^{n+1} + (\mathbf{E}_1)_i^n \left[\frac{(\Delta T_\tau)_{i+1}^{n+1} - (\Delta T_\tau)_{i-1}^{n+1}}{2\Delta x}\right] + (\mathbf{E}_{11})_i^n \left[\frac{(\Delta T_\tau)_{i+1}^{n+1} - 2(\Delta T_\tau)_i^{n+1} + (\Delta T_\tau)_{i-1}^{n+1}}{\Delta x^2}\right] = -\mathbf{Q}_i^n$$

$$\underbrace{\left[\frac{(\mathbf{E}_1)_i^n}{2\Delta x} + \frac{(\mathbf{E}_{11})_i^n}{\Delta x^2}\right]}_{\mathbf{A}_i^n} (\Delta T_\tau)_{i+1}^{n+1} + \underbrace{\left[1 - \frac{2(\mathbf{E}_{11})_i^n}{\Delta x^2}\right]}_{\mathbf{B}_i^n} (\Delta T_\tau)_i^{n+1} +$$

$$\underbrace{\left[\frac{(\mathbf{E}_{11})_i^n}{\Delta x^2} - \frac{(\mathbf{E}_1)_i^n}{2\Delta x}\right]}_{\mathbf{C}_i^n} (\Delta T_\tau)_{i-1}^{n+1} = -\mathbf{Q}_i^n,$$

$$\mathbf{A}_i^n\ (\Delta T_\tau)_{i+1}^{n+1} + \mathbf{B}_i^n\ (\Delta T_\tau)_i^{n+1} + \mathbf{C}_i^n\ (\Delta T_\tau)_{i-1}^{n+1} = -\mathbf{Q}_i^n \quad (11)$$

With

$$(\mathbf{E_1})_i^n = (\Delta t s_3\, \mathbf{b}_i)^n \quad (12a)$$

$$(\mathbf{E_{11}})_i^n = \left(\Delta t s_3 \mathbf{c}_{11} - \frac{\Delta t^2}{2} s_4 (\mathbf{b}_1)^2\right)_i^n \quad (12b)$$

$$(\mathbf{G_K})_i^n = -k\left[\frac{(T_\mathbf{K})_{i+1}^n - (T_\mathbf{K})_{i-1}^n}{2\Delta x}\right] \quad (12c)$$

$$(\mathbf{b_1})_i^n = \frac{1}{\rho c}\left(\frac{\partial \mathbf{G_K}}{\partial \mathrm{T}_\tau}\right)_i^n = \frac{1}{\rho c}\left[\frac{\mathbf{G_K}_i^n - \mathbf{G_K}_i^{n-1}}{\mathrm{T_\tau}_i^n - \mathrm{T_\tau}_i^{n-1}}\right] \quad (12d)$$

$$(\mathbf{c}_{11})_i^n = \frac{1}{\rho c}\left(\frac{\partial \mathbf{G_K}}{\partial \mathrm{T}_{\tau,i}}\right)_i^n =$$

$$\frac{1}{\rho c}\left[\frac{\mathbf{G_K}_i^n - \mathbf{G_K}_i^{n-1}}{\left(\frac{\mathrm{T}_{\tau\, i+1} - \mathrm{T}_{\tau\, i-1}}{2\Delta x}\right)^n - \left(\frac{\mathrm{T}_{\tau\, i+1} - \mathrm{T}_{\tau\, i-1}}{2\Delta x}\right)^{n-1}}\right] \text{ \& or}$$

$$\frac{1}{\rho c}\left[\frac{\mathbf{G_K}_i^n - \mathbf{G_K}_i^{n-1}}{\left(\frac{\mathrm{T}_{\tau i+1} - \mathrm{T}_{\tau\, i}}{\Delta x}\right)^n - \left(\frac{\mathrm{T}_{\tau\, i+1} - \mathrm{T}_{\tau\, i}}{\Delta x}\right)^{n-1}}\right] \quad (12e)$$

$$\mathbf{Q}_i^n = \frac{\Delta t}{\rho c}\left(\frac{\mathbf{G_K}_{i+1} - \mathbf{G_K}_{i-1}}{2\Delta x}\right)^n - \frac{\Delta t^2}{2\rho c}\,\mathbf{b}_i^n\left(\frac{\mathbf{G_K}_{i+1} - 2\mathbf{G_K}_i + \mathbf{G_K}_{i-1}}{\Delta x^2}\right)^n \quad (12f)$$

$$(s)_i^n = \left(s_{Spatiotemporal}\right)_i^n = \frac{\left(s_{Spatial}\right)_i^n + \left(s_{Temporal}\right)_i^n}{2} \quad (12g)$$

$$\left(s_{Spatial}\right)_i^n = \frac{\sqrt{\boldsymbol{max}(\mathrm{T}_{i-1}^n, \mathrm{T}_{i+1}^n)^2 - \boldsymbol{min}(\mathrm{T}_{i-1}^n, \mathrm{T}_{i+1}^n)^2}}{\boldsymbol{min}(\mathrm{T}_{i-1}^n, \mathrm{T}_{i+1}^n)} \quad (12h)$$

$$\left(s_{Temporal}\right)_i^n = \frac{\sqrt{\boldsymbol{max}(\mathrm{T}_i^{n-1}, \mathrm{T}_i^n)^2 - \boldsymbol{min}(\mathrm{T}_i^{n-1}, \mathrm{T}_i^n)^2}}{\boldsymbol{min}(\mathrm{T}_i^{n-1}, \mathrm{T}_i^n)} \quad (12i)$$

$$(s_3)_i^n = \begin{cases} \min((s)_i^n, 1) & (s)_i^n > \omega \quad (\omega < 1) \\ 0 & (s)_i^n < \omega \quad \mathrm{T_{min}} \neq 0 \end{cases} \quad (12j$$

$$(s_4)_i^n = [f\,(s_3, \eta)]_i^n \text{ or } = \frac{1}{2}\{1 + [(s_3)_i^n]^\eta\} \quad (12k)$$

In Eq. (12h) and Eq.(12i), the case of $T_{min} = 0$ can never arises as we here always take the temperature in Kelvin scale only. ω is user defined specified small number less than 1.For the value of ω and range for η refer [5].

In Eq.(11) combined with Eq.(12) , because of the three time levels in this FDV method, initial data must be known at two time levels i.e. n & n-1 time levels. These data can be determined if the time derivative of temperature is specified at t=0. The finite-differenced FDV Eq. (11) on applying to the grid points in a one-dimensional computational domain combined with two initial & two boundary conditions, results into a system of linear, algebraic equations which can be solved using standard algorithm of matrix solver to compute $(\Delta T_\tau)_i^{n+1}$ variables at all grid points in the domain for time level n+1. Thomas algorithm of tri-diagonal matrix solver can be used if tri-diagonal system of linear, algebraic equations is generated at each time step. Whichever algorithm of matrix solver is used, it has to modified to include the provision of updating $\mathbf{A}_i^n$, $\mathbf{B}_i^n$, $\mathbf{C}_i^n$ & $\mathbf{Q}_i^n$ in Eq.(11) at each subsequent time steps. Finally we decode computed element $(\Delta T_\tau)_i^{n+1}$ to obtain the primitive temperature solution variables T(x, t) =T as in Eq.(13):

$$\mathrm{T}_i^{n+1} = \frac{1}{\left(1+\frac{\tau}{\Delta t}\right)}\left[(\Delta \mathrm{T}_\tau)_i^{n+1} + \left(1 + \frac{2\tau}{\Delta t}\right)\mathrm{T}_i^n - \frac{\tau}{\Delta t}\mathrm{T}_i^{n-1}\right] \quad (13)$$

Thus from Eq.(11) / Eq.(13) based on FDV method , primitive temperature solution of a one dimensional non-Fourier heat conduction of Jeffrey's type is computed that is repeated for each time step as the heat wave proceeds through the still constant property fluid or the solid medium with a constant speed 'a'. Further it is possible to upgrade the finite differenced FDV Eq. (11) for 3-D non-Fourier heat conduction problems in still constant property fluid or in solid.

3. RESULTS AND DISCUSSION

As a numerical example, consider 1-D Jeffrey's ($\mathbf{K} \neq 0$) non-Fourier heat transport effects in a purely heat conducting medium of a still thin film of constant property nano-fluid [11] comprising of water as base fluid solvent and Titanium dioxide (TiO_2) as solute nano-particle and assume that both the base fluid and nano-particles are in local thermal equilibrium during the numerical simulation. Initially the nano-fluid is at temperature T_0. At the time t >0, both the end surfaces of the thin nano-fluid film at x=0 and x=l are impulsively [12] increased to a temperature T_w (*that is a spatial temperature gradient is suddenly applied at both ends*) and this sets up Jeffrey's (F_T = 0.86) non-Fourier transient temperature distributions in the assumed purely heat conduction nano-fluid medium. Thus the solution of the finite-differenced FDV Eq. (11) characterizing this numerical problem of one dimensional Jeffrey's type (F_T = 0.86) non-Fourier heat transfer in the nano-fluid can be performed with following initial & boundary conditions and dimensional- less variables:

$$\mathrm{T}(x, t) = \mathrm{T}_0 = 298\text{ K } \& \frac{\partial\,[\mathrm{T}(x,0)]}{\partial t} = 0 \text{ for t=0} \quad (14)$$

$$\mathrm{T}(0, t) = \mathrm{T}(l, t) = \mathrm{T}_w = 323\mathrm{K} \text{ for t>0} \quad (15)$$

Dimensionless variables:

$$\mathrm{T}^* = \frac{\mathrm{T}(x,t) - \mathrm{T}_0}{\mathrm{T}_w - \mathrm{T}_0};\; x^* = \frac{x}{2\sqrt{\tau\alpha}};\; t^* = \frac{t}{2\tau} \quad (16)$$

As discussed earlier, because of the three time levels in this FDV method, initial data must be known at two time levels i.e. n-1 & n time levels. These data can be determined if the time derivative of temperature is specified at t=0. That is for the first time step Δt, based on zero valu e of second order central finite difference expression of time derivative of temperature as given in Eq. (14) we obtain an initial condition as

$$\frac{\partial\,[\mathrm{T}(x,0)]}{\partial t} = 0 \therefore \frac{\mathrm{T}(x,0)^{n+1} - \mathrm{T}(x,0)^{n-1}}{2\Delta t} \approx 0$$

$$\rightarrow \mathrm{T}(x,0)^{n-1} = \mathrm{T}(x,0)^{n+1} \rightarrow \mathrm{T}_{\mathbf{i}}^{\mathbf{n-1}} = \mathrm{T}_{\mathbf{i}}^{\mathbf{n+1}} \quad (17)$$

The finite differenced expression for 1-D Jeffery's type non-Fourier heat conduction Eq. (1) can be written as:

$$-\frac{\mathbf{K\lambda}}{\Delta t} T_{i-1}^{n+1} + \left(1 + \frac{\Delta t}{\tau} + \frac{2\,\mathbf{K\lambda}}{\Delta t}\right) T_i^{n+1} - \frac{\mathbf{K\lambda}}{\Delta t} T_{i+1}^{n+1} = -T_i^{n-1} + \left(\lambda - \frac{\mathbf{K\lambda}}{\Delta t}\right) T_{i-1}^{n} + \left(2 + \frac{\Delta t}{\tau} - 2\lambda + \frac{2\,\mathbf{K\lambda}}{\Delta t}\right) T_i^{n} + \left(\lambda - \frac{\mathbf{K\lambda}}{\Delta t}\right) T_{i+1}^{n} \quad (18)$$

Substituting the initial condition Eq. (17) in Eq. (18), we obtain the finite differenced expression for first time step Δt as:

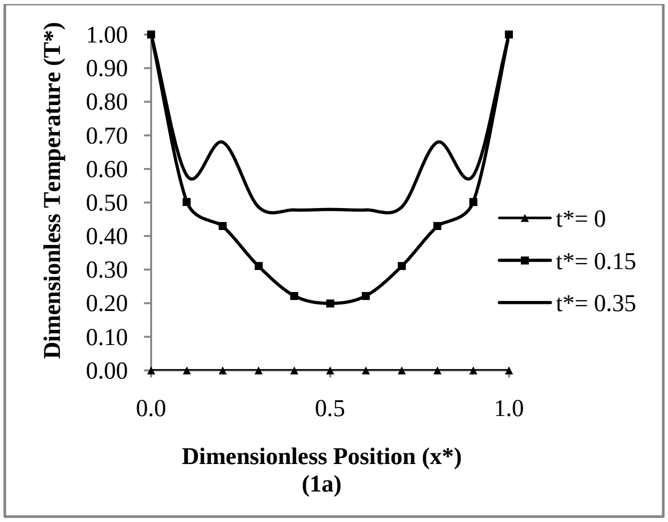
$$-\frac{\mathbf{K\lambda}}{\Delta t} T_{i-1}^{n+1} + \left(2 + \frac{\Delta t}{\tau} + \frac{2\,\mathbf{K\lambda}}{\Delta t}\right) T_i^{n+1} - \frac{\mathbf{K\lambda}}{\Delta t} T_{i+1}^{n+1} = \left(\lambda - \frac{\mathbf{K\lambda}}{\Delta t}\right) T_{i-1}^{n} + \left(2 + \frac{\Delta t}{\tau} - 2\lambda + \frac{2\,\mathbf{K\lambda}}{\Delta t}\right) T_i^{n} + \left(\lambda - \frac{\mathbf{K\lambda}}{\Delta t}\right) T_{i+1}^{n} \quad (19)$$

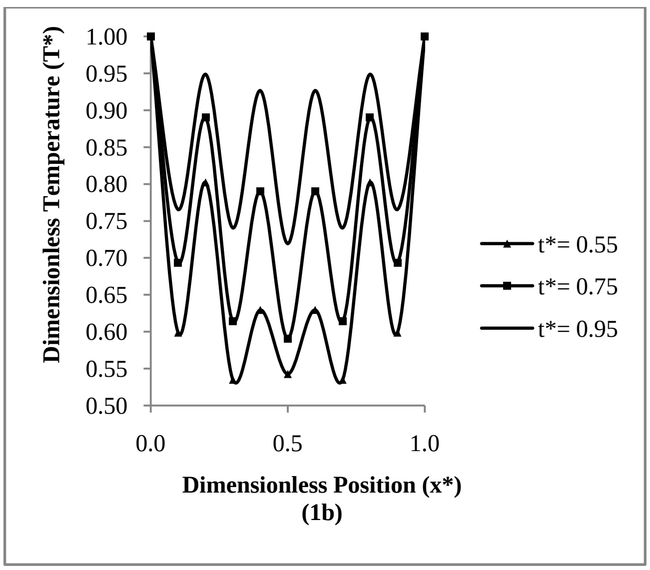
On applying the Eq. (19) to the all intermediate grid points , we obtain a tri-diagonal system of linear, algebraic equations whose solution by means of Thomas' Algorithm results in the values of the unknown temperatures at all the intermediate grid points of 1-D domain at time t=Δt. As a result of Eq. (14), Eq. (15) and Eq. (19), we have known initial temperature data at two time levels i.e. n-1 (t=0) & n (t=Δt) time levels. Based on these initial temperature data at (n-1) & n time levels and Eq.(16), we now proceed to apply our derived FDV Eq.(11) along with Eq.(12a) to Eq.(12k) to all the intermediate grid points of 1-D domain for t= 2Δt. This leads to a tri-diagonal system of linear, algebraic equations with unknowns ΔT_τ at time level (n+1) for all the intermediate points of 1-D domain. Using Thomas' Algorithm as standard for the treatment of the generated tri-diagonal systems of equations, we compute the values of $(\Delta T_\tau)_i^{n+1}$ at all the intermediate grids. Finally the primitive transient temperature solution variables T_i^{n+1} at various intermediate grid points for (n+1) time level are obtained by substituting the computed element $(\Delta T_\tau)_i^{n+1}$ in Eq. (13).

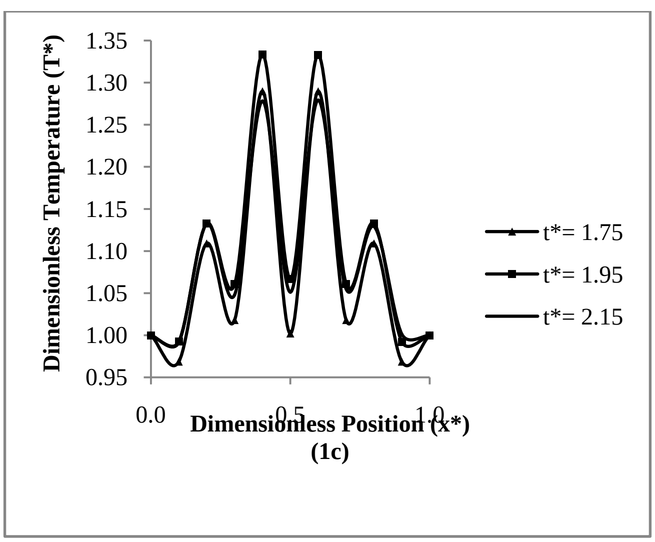
From the starting time t=0, final results of spatial dimensionless temperature distributions based on Eq. (16) at different instants of dimensionless time predicted by the FDV model are presented in Figure.1(a)-(d). The FDV model upholds the existence of heat waves in the thin nano-fluid and demonstrates the propagation process of heat waves, the magnitude and profile of transient temperature. This brings FDV model in par with other existing numerical models for non-Fourier heat conduction simulation. Once the temperature at thin film end boundaries are spontaneously raised, the film temperature is increased from T_o as time marches and there is the region of the temperature intensification and it's no effect region in the film. That is the temperature is propagated through the film with a finite speed contrasting the Fourier's law of infinite speed which is physically inadmissible. Superposition of the moving left and right heat waves results in wave interference phenomena [13] leading to incidents of rise in temperature at the intermediate grid points of the nano-fluid film above the imposed temperature at both the boundary. Further more such peak temperature effects have been explained by the extended irreversible thermodynamics theory [14] theory. Figure.1(c)-(d). illustrates such temperature overshooting incidents. Finally it can be observed in Fig. 1(d), the oscillating peak temperature dies off for the dimensionless time t*=6.50 denoting t=2.665 ms, that is the numerical solution is converged, the final steady state temperature distribution conditions are reached with temperature filling the whole nano-fluid film at T(x, t ≥ 2.665) = T_w (=323 K).

In the above results and discussion for the first time step Δt in the R.H.S of Eq. (19), the temperature values *at all the grid points* of 1-D domain were known from the initial conditions Eq. (14) i.e. T_0 . That is the time starts (t=0) when the temperature at the both boundaries of the thin film are at T_0 (=298 K). Now let us carry out Jeffrey's non-Fourier heat transfer numerical simulation for the case where the time starts (t >0) when the temperature at the both boundaries of the thin film switches to T_w (=323 K) and then keeps constant. From the starting time t > 0, final results of spatial dimensionless temperature distributions based on Eq. (16) at different instants of dimensionless time predicted by the FDV model are presented in Figure.2(a)-(d). Significant heat wave nature of the temperature distributions can be observed from Figure.2 (b) onwards. As the time progresses, wavy nature of the temperature propagation diminishes. Lastly the numerical solution is converged, the final steady state temperature distribution conditions are reached at t*=4.00 denoting t=1.64 ms with temperature filling the whole nano-fluid film at T(x, t ≥ 1.64) = T_w (=323 K).

Numerical results of the manner in which the Heat Conduction Model Number 'F_T' affects the Jeffrey's non-Fourier temperature behavior is next illustrated in the Figure.3 (a)-(d). Here the value of τ is maintained constant but the values of '**K**' are increased. Figure.3 (a)-(d), it can be observed that the wave nature of the Jeffrey's non-Fourier temperature distribution for small value of F_T = 0.01 is highly significant and its enhanced temperature overshooting phenomena can be viewed uniquely in Fig.3 (b). Throughout the Fig. (3), it can be seen that the increase of 'F_T' i.e. '**K**' leads to stronger non-Fourier heat diffusion and weaker non-Fourier heat wave propagation. Greater the 'F_T' i.e. phase lag for temperature gradient **'K',** the more even the temperature distribution will be. Physically the sharp wave fronts due phase lag for heat flux 'τ' can be smoothened effectively by increasing phase lag for temperature gradient '**K**' leading to non-Fourier diffusion like heat conduction.







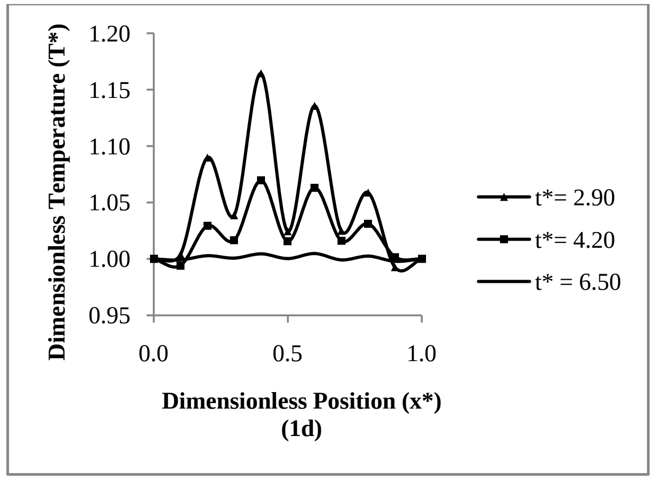
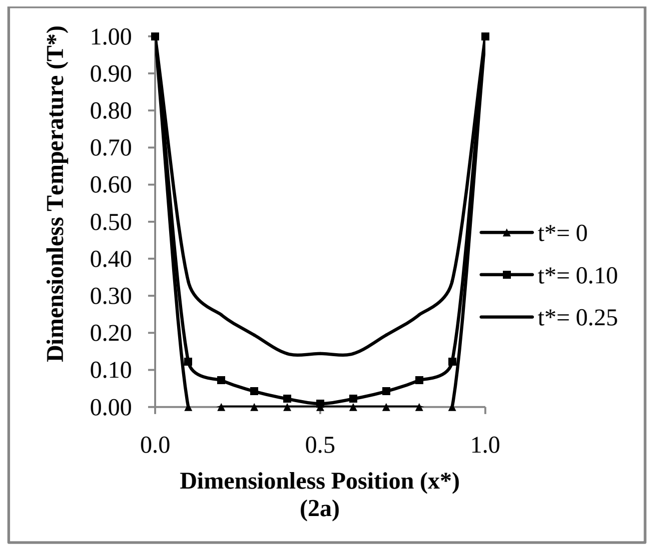
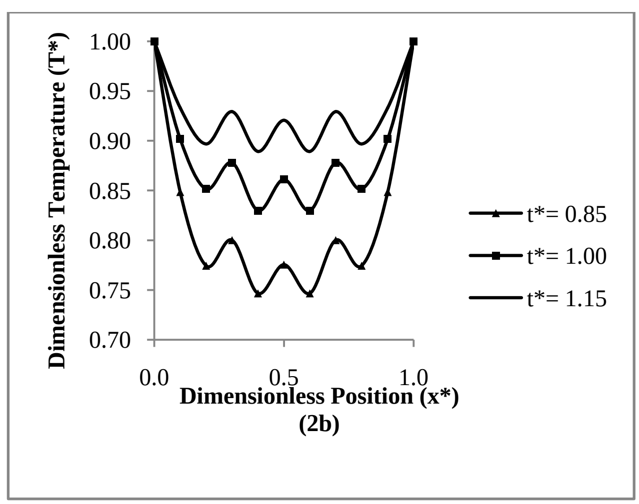
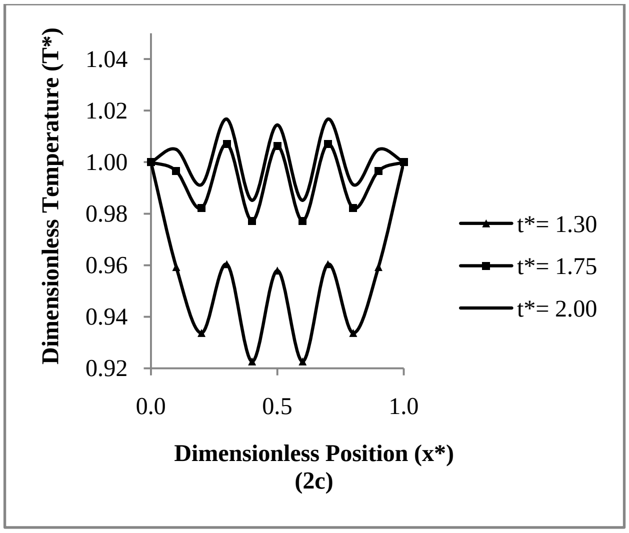


Fig. 1: Jeffrey's non-Fourier heat conduction solutions from the starting time t=0 for F_T = 0.86 predicted by FDV method at different instants of time (a) t*=0, 0.15, 0.35 (b) t*= 0.55, 0.75, 0.95(c) t*=1.75, 1.95, 2.15 (d) t*=2.90, 4.20, 6.50







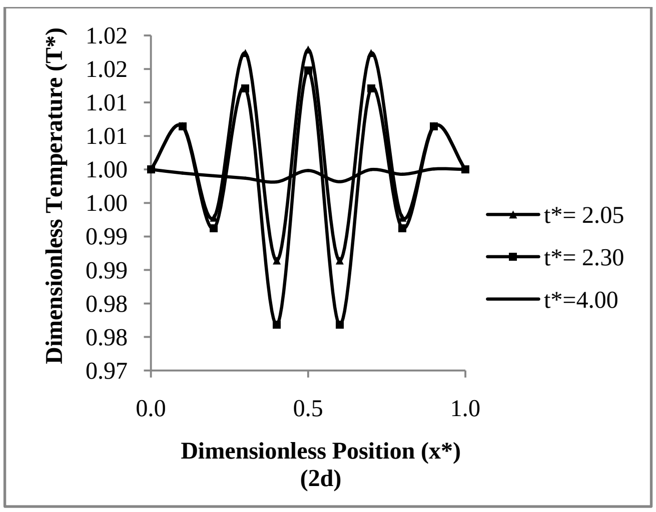
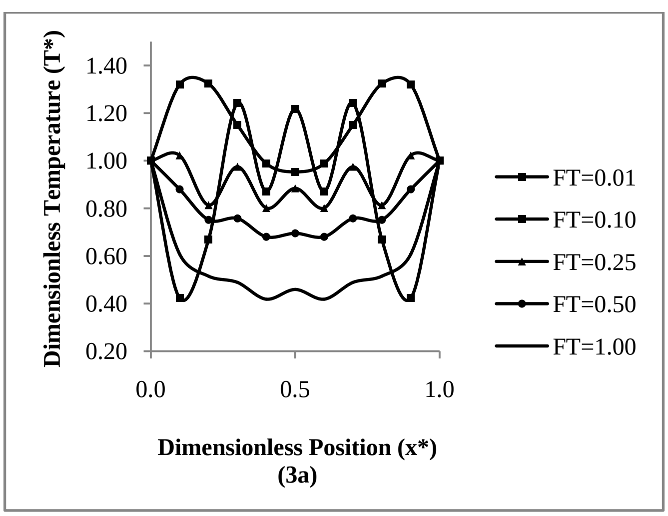
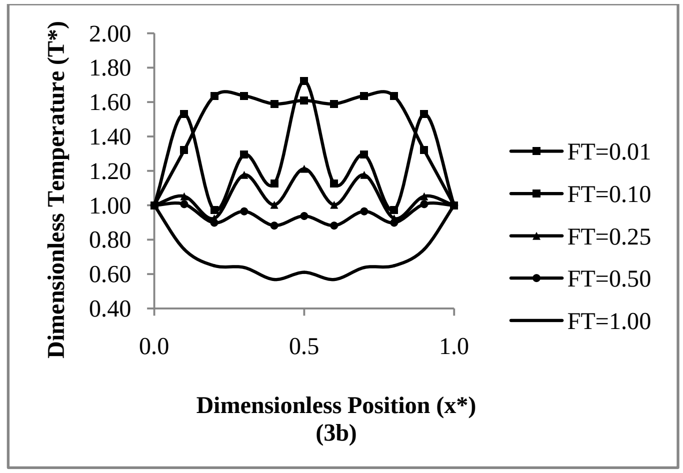
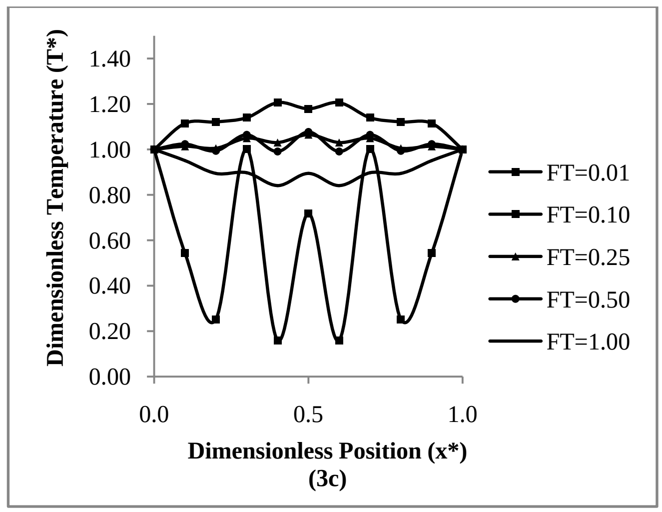


Fig. 2: Jeffrey's non-Fourier heat conduction solutions from the starting time t > 0 for F_T = 0.86 predicted by FDV method at different instants of time (a) t*=0, 0.10, 0.25 (b) t*= 0.85, 1.00, 1.15(c) t*=1.30, 1.75, 2.00 (d) t*=2.05, 2.30, 4.00







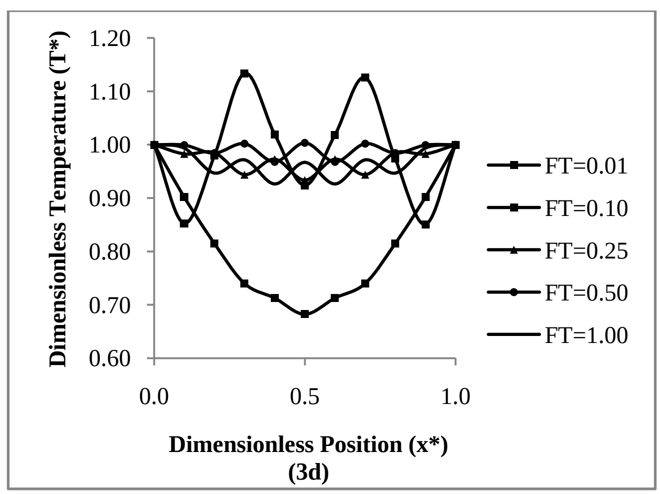


Fig. 3: Diffusion effects of increasing the values of Heat Conduction Model Number 'F_T' on the Jeffrey's non-Fourier temperature distributions as predicted by FDV method at different instants of time (a) t*=0.60 (b) t*=0.8 (c) t*=1.65 (d) t*=2.25

Similar results for various initial and boundary conditions were established by the former investigators [8, 15-19]. This show the validity and credibility of our derived FDV algorithm for Jeffrey's non-Fourier heat conduction as it has the intrinsic ability to smoothen Cattaneo's non-Fourier sharp heat waves embedded in the Jeffrey's model on increasing the time lag of the temperature gradient '**K'.** Further from the Figure.3 (a)-(d), it can be observed that temperature distributions for $F_T = 1$ i.e. $\mathbf{K} = \tau$ were approaching the classical Fourier heat diffusion solution and such similar results can be seen in [17-19].

4. CONCLUSION

The transient temperature distribution simulation in a nano-fluid film subjected to an impulsive boundary temperature conditions using FDV model revealed a finite heat wave speed in the heat conduction process in contrast with the classical Fourier heat conduction. Consequently a finite difference / finite element scheme based on FDV methodology can now serve as an alternative to the existing numerical and analytical methods for non-classical heat transfer of Jeffrey's type.

The exceptionality of this FDV algorithm is that for every time step, coefficients ($\mathbf{A}_i^n$**,** $\mathbf{B}_i^n$ **&** $\mathbf{C}_i^n$) of Eq. (11) will change as the local temperature field changes and will modify the governing FDV Eq. (11) to solve the appropriate physics of hyperbolic, parabolic or mixed nature that are going on at each grid point. This is in contrast with other existing numerical schemes where normally such coefficients [L.H.S coefficients of Eq. (18)] expressed in terms of the conducting medium's thermo-physical properties (α, τ & **K**), computational spatial & time increments (Δx & Δt) remains constant.

The parametric study revealed that the effects of phase lag '**K**' of the temperature gradient on the thermal behavior are significant in the Jeffrey's non-Fourier heat conduction. Quantitatively the increase in Heat Conduction Model Number 'F_T' resulted in smoothening the Cattaneo's non-Fourier sharp heat waves embedded in the Jeffrey's model leading to non-Fourier diffusion like heat conduction. Furthermore it was observed for $F_T = 1$ i.e. for all $\mathbf{K} = \tau$, the temperature distributions were approaching the classical Fourier heat diffusion solution. The above two similar results were also established by the former investigators and this show the validity and credibility of our derived FDV algorithm for Jeffrey's non-Fourier heat transfer.

Present work was confined to non-Fourier heat conduction in still fluid. Future broad research will be required to develop FDV formulation for numerical simulation of 3-D non-Fourier heat transfer problems both in flowing Newtonian & non-Newtonian fluids also.

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NOMENCLATURE

U conservation variable $\mathbf{G}_i$ diffusion flux variables

Δx spatial increment $\mathbf{U}_\tau$ $\mathbf{U}(x, t + \tau)$

Δt time increment T_τ $T(x, t + \tau)$

k thermal conductivity $\mathbf{G}_\mathbf{K}$ $\mathbf{G}(x, t + \mathbf{K})$

T temperature T(x, t) $T_\mathbf{K}$ $T(x, t + \mathbf{K})$

c specific heat ΔT_τ^{n+1} $T_\tau^{n+1} - T_\tau^n$

k_1 effective thermal conductivity

k_2 elastic conductivity ($k = k_1 + k_2$)

a Heat wave speed $(\frac{\alpha}{\tau})^{1/2}$

K time parameter $\tau\,(\frac{k_1}{k})$

$\mathbf{F}_i$ convection flux variables

s FDV parameter

s_3 1st order diffusion FDV parameter

s_4 2nd order diffusion FDV parameter

F_T Heat Conduction Model No. $(\frac{\mathbf{K}}{\tau})$

$\mathbf{b}_i$ diffusion Jacobian

$\mathbf{c}_{ij}$ diffusion gradient Jacobian

l heat conducting medium thickness

Greek symbols

τ thermal relaxation time   ρ mass density

α thermal diffusivity $(\frac{k}{\rho c})$ λ $\frac{\alpha \Delta t^2}{\tau \Delta x^2}$

ω user defined specified small number

η Number relating s_3 and s_4

Superscript

n-1, n, n+1 running index in the time direction

***** dimensionless parameter

Subscript

i, j co-ordinate dimension counters =1, 2, 3

in a partial differential equation

i-1, i, i+1 running index in the x direction in a finite

differenced equation

0 initial

w boundary surface